Towards a self-consistent modeling of anomalous transport in fluid simulations of Hall Effect Thrusters

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Laboratoire de Physique des Plasmas

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Summary

I. 1D fluid model

II. Comparison between ad-hoc and anomalous transport

I) 1D fluid model

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Fluid model with ad-hoc mobility

One-dimensional (axial direction) fluid model[1] with the following assumptions:

- Quasi-neutrality;
- No electron inertia;
- Cold ions;
- Isotropic electron distribution.

$$\frac{\partial n_g}{\partial t} + v_g \frac{\partial n_g}{\partial z} = -n_g n_i K_{iz} + \nu_{iw} n_i$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial (v_i n_i)}{\partial z} = n_g n_i K_{iz} - \nu_{iw} n_i$$

$$\frac{\partial (n_i v_i)}{\partial t} + \frac{\partial}{\partial z}\left(n_i v_i^2 + n_i c_s^2\right) = n_g n_i v_g K_{iz} - \nu_{iw} n_i v_i - \frac{q}{M}\frac{n_i v_i - I_0}{\mu}$$

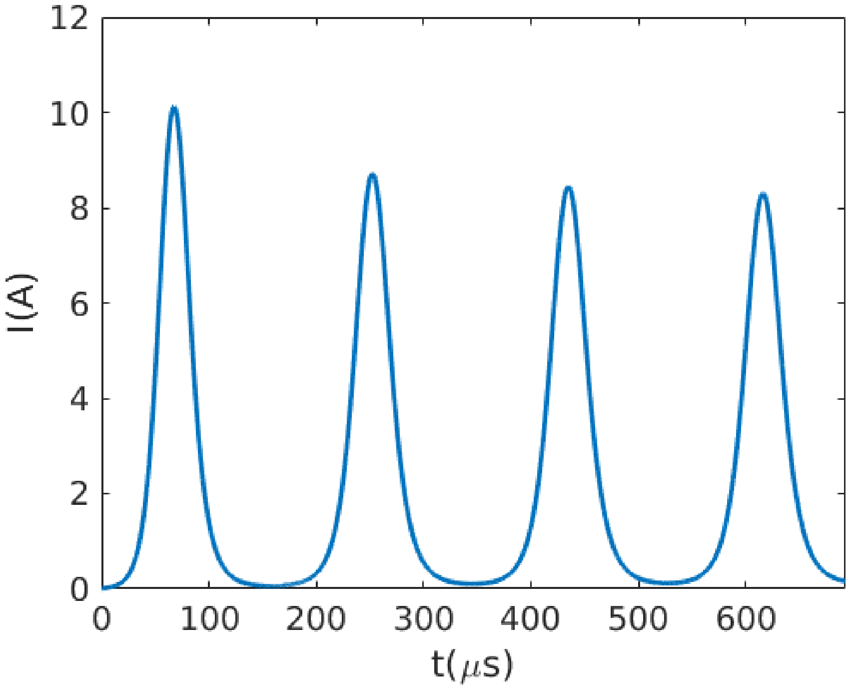
$$\frac{\partial}{\partial t}\left(\frac{3}{2} n_i T_e\right) + \frac{\partial}{\partial z}\left(\frac{3}{2} n_i T_e v_i\right) = \frac{\partial}{\partial z}\left(\frac{3}{2} T_e I_0\right) - n_i T_e \frac{\partial v_{ez}}{\partial z} - \nu_{iw} n_i \epsilon_w - n_i n_g \gamma \epsilon_i K_{iz}$$

$$\nu_m = n_g K_{el} + \nu_{ew} + \alpha_B \omega_{ce}$$ ► Bohm-like anomalous transport

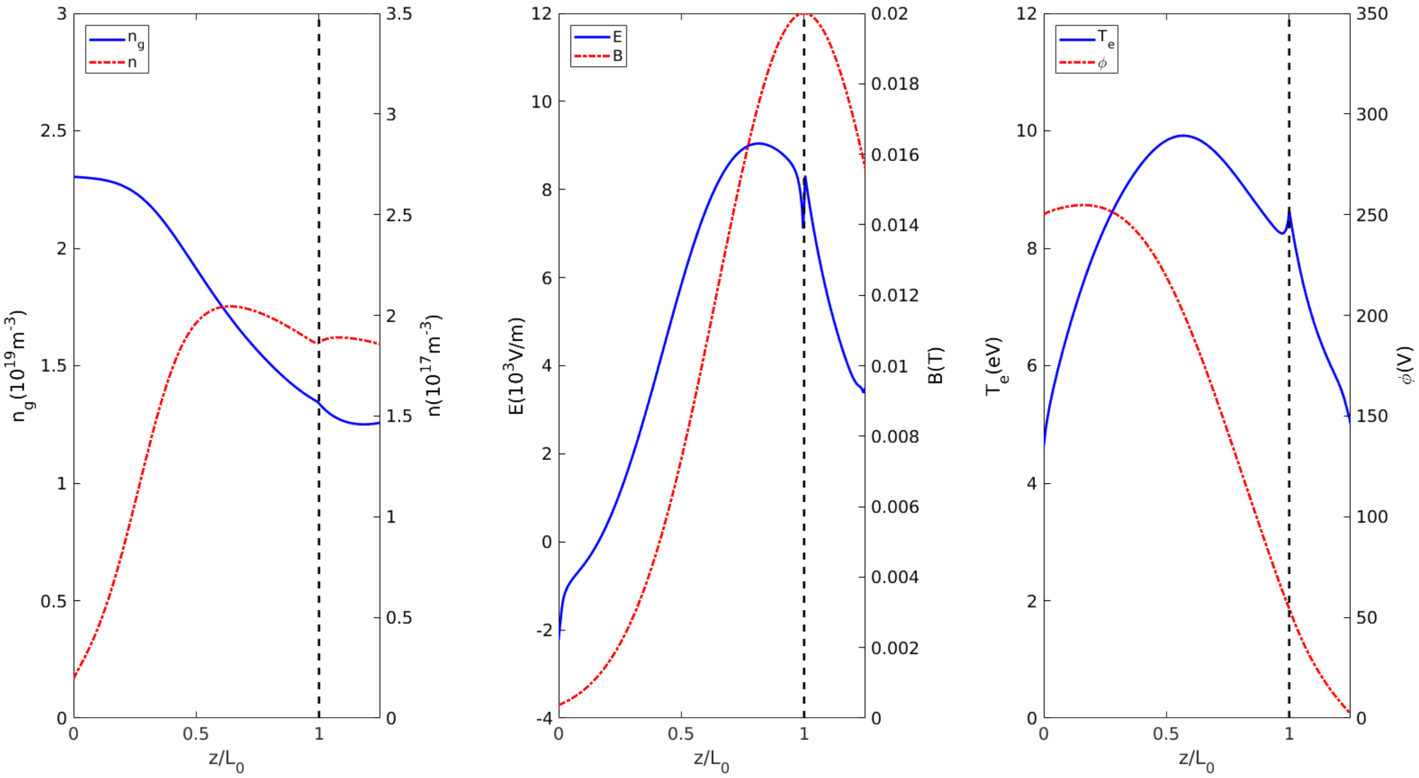
[1] S. Barral, and E. Ahedo, Low-frequency model of breathing oscillations in Hall discharges, Phys. Rev. E. 79 046401 (2009)

Results for SPT-100 at 250 V

Parameter	**Value**
Outer radius (R_2)	$5\ cm$
Inner radius (R_1)	$3\ cm$
Length of the thruster (L_0)	$4\ cm$
Length of the simulation box (L)	$5\ cm$
Applied voltage (V_0)	$250\ V$
Maximum magnetic field (B_0)	$200\ G$
Mass flow rate ($\dot{m}$)	$5\ mg/s$
Initial gas velocity (v_{g0})	$200\ m/s$
Initial electron temperature (T_{e0})	$5\ eV$
Ion temperature (T_{i0})	$1\ eV$
Ionization energy (ϵ_i)	$12.1\ eV$
Effective ionization cost factor (γ_i)	3
Anomalous collision factor (α_B)	$\frac{1}{160}$



Average plasma characteristics



II) Comparison between ad-hoc and anomalous transport

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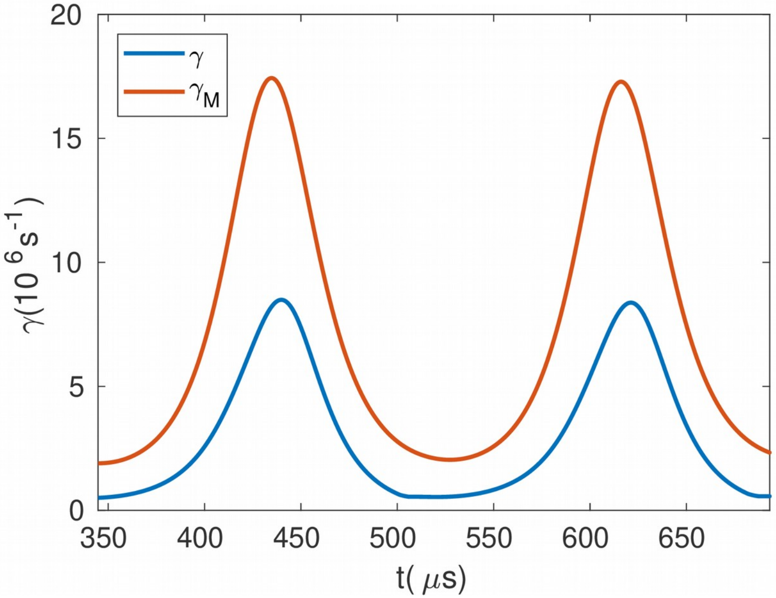
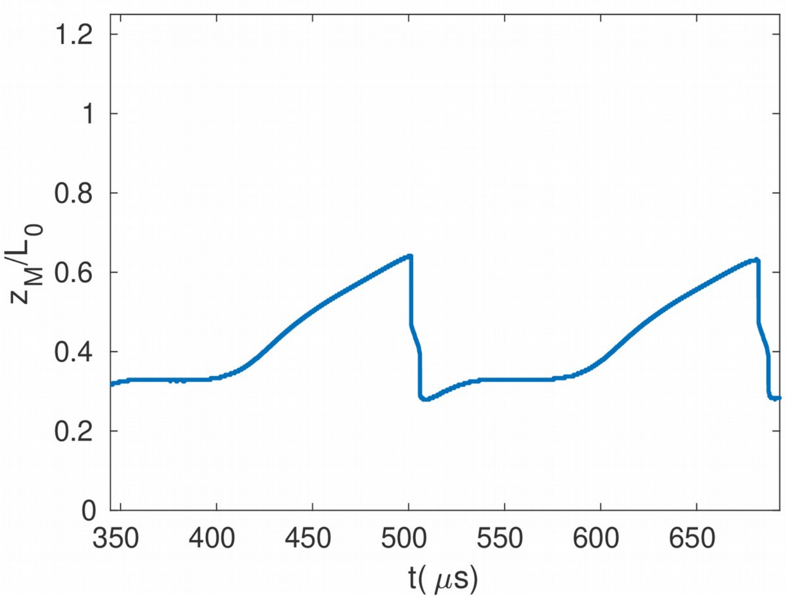
Characteristics of the instability

$$\gamma \simeq -\sqrt{\frac{\pi}{8}}\frac{kc_s}{(1+k^2\lambda_D^2)^2}\left\{\left(\frac{T_e}{T_i}\right)^{\frac{3}{2}} e^{-\frac{T_e}{2T_i}\frac{1}{1+k^2\lambda_D^2}} + \sqrt{\frac{m_e}{m_i}}\left[1 \mp \frac{\mathbf{k}\cdot\Delta\mathbf{v}_d}{kc_s}\sqrt{1+k^2\lambda_D^2}\right]\right\}$$

$$\gamma_M \simeq \sqrt{\frac{\pi m}{54M}}\frac{v_{de}}{c_s}\omega_{pi}$$





- The characteristics of the instability are affected by the large scale oscillations;
- The maximum growth rate is a too severe approximation.

Implementing instability-induced anomalous transport

Friction force and anomalous heating from quasi-linear theory

$$\mathbf{R}_{ei} = q_e \langle \delta n \delta E_\theta \rangle \longrightarrow R_{ei} = 2\int dk_y \mathcal{E}_k k_y \Im[\chi_e(\omega_R + i\gamma, k_y)]$$

$$S_{ei} = \mathbf{R}_{ei} \cdot \mathbf{v}_e$$

Evolution of the spectral energy density

$$\frac{\partial \mathcal{E}}{\partial t} + \nabla_{\mathbf{x}} \cdot (\langle \mathbf{v}_g \rangle_k \mathcal{E}) = 2\langle \gamma \rangle_k \mathcal{E}$$

$$\langle f(\mathbf{k}, \mathbf{x}, t) \rangle_k = \frac{\int dk f(\mathbf{k}, \mathbf{x}, t)\mathcal{E}_k}{\int dk \mathcal{E}_k}$$

Spectral energy density peaked around the maximum growth rate[1]

$$\langle F(\mathbf{k}, \mathbf{x}, t) \rangle_k = \frac{1}{2}(F(\mathbf{k}_M, \mathbf{x}, t) + F(-\mathbf{k}_M, \mathbf{x}, t))$$

$$\langle \gamma \rangle_k = \gamma_M - \frac{\sqrt{\pi}}{9}\omega_{pi}\left(\frac{T_e}{T_i}\right)^{\frac{3}{2}} e^{-\frac{T_e}{3T_i}} \Longrightarrow R_{ei} = -2\frac{\sqrt{2\pi}}{\lambda_{De}}\frac{v_e}{v_{Te}}\mathcal{E}$$

[1] R. C. Davidson, and N. A. Krall, Anomalous transport in high-temperature plasmas with applications to solenoidal fusion systems, Nucl. Fusion 17 1313 (1977)

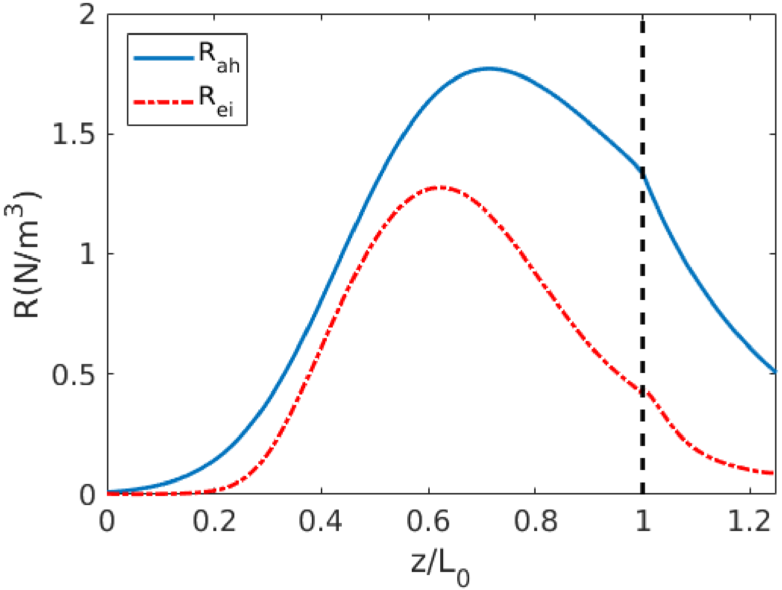
Comparison of friction forces and heating

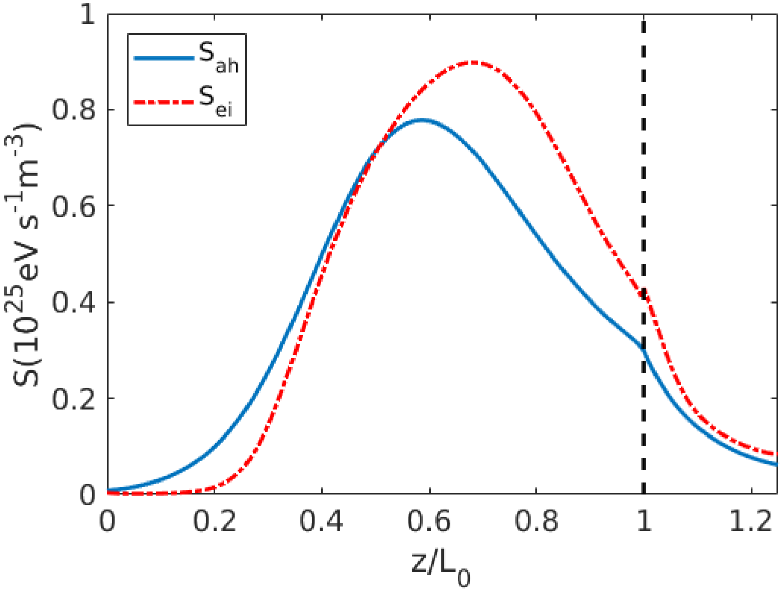
$$R_{ah} = -mn\alpha_B \omega_{ce} v_{ey}$$

$$R_{ei} = -2\frac{\sqrt{2\pi}}{\lambda_{De}} \frac{v'_{ey}}{v_{Te}} \mathcal{E}$$

$$S_{ah} = mn\alpha_B \omega_{ce} v_{ey}^2$$

$$S_{ei} = R_{ei} v'_{ey}$$





Good agreement between the ad-hoc anomalous transport and the instability-induced one

Conclusion

- Good agreement between the ad-hoc model and the anomalous transport induced by the instability for the same parameters of the plasma.

On the other hand, implementing self-consistently the anomalous transport in the simulation does not gives the desired results.

Possible reasons:

- Full solution of the dispersion relation required;
- Better inclusion of the nonlinear effects of the instability, especially the evolution of the distribution function and its effects on the growth rate[1];
- Including an equation for the ion temperature.

[1] T. Lafleur, R. Martorelli, P. Chabert, A. Bourdon, Anomalous electron transport in Hall-effect thrusters: Comparison between quasi-linear kinetic theory and particle-in-cell simulations, Phys. Plasmas 25 061202 (2018)

Thanks for your attention !

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Annexes

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A) Details on wave kinetic equation

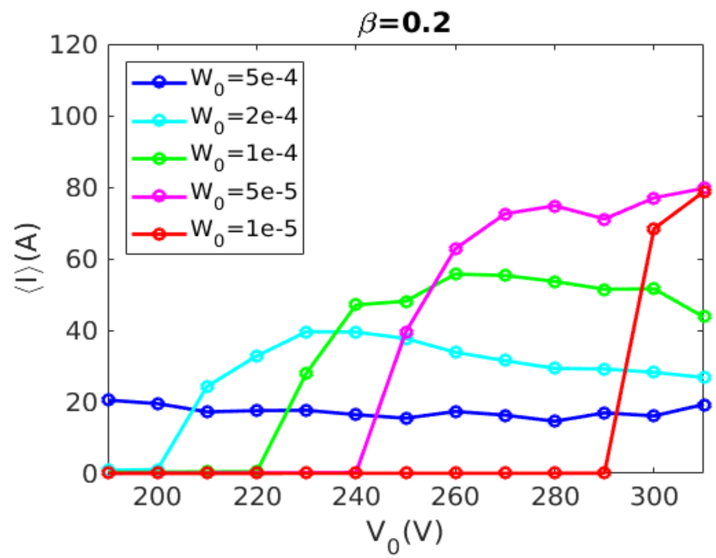
$$\mathcal{E}_0 = \frac{qT_e}{4\lambda_{De}^3}$$

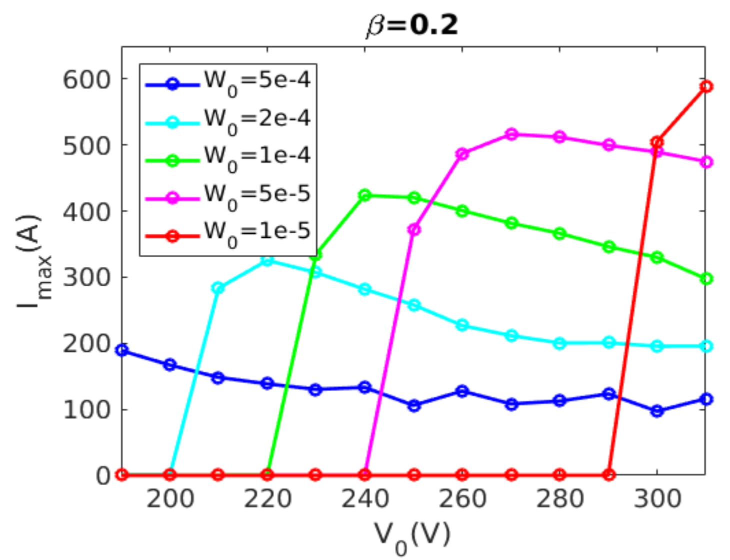
Initial value for energy density: energy density associated to thermal fluctuations in the case of ion-acoustic instability

Maximum value of the energy density is set by ion-trapping condition

B) Results for self-consistent simulations

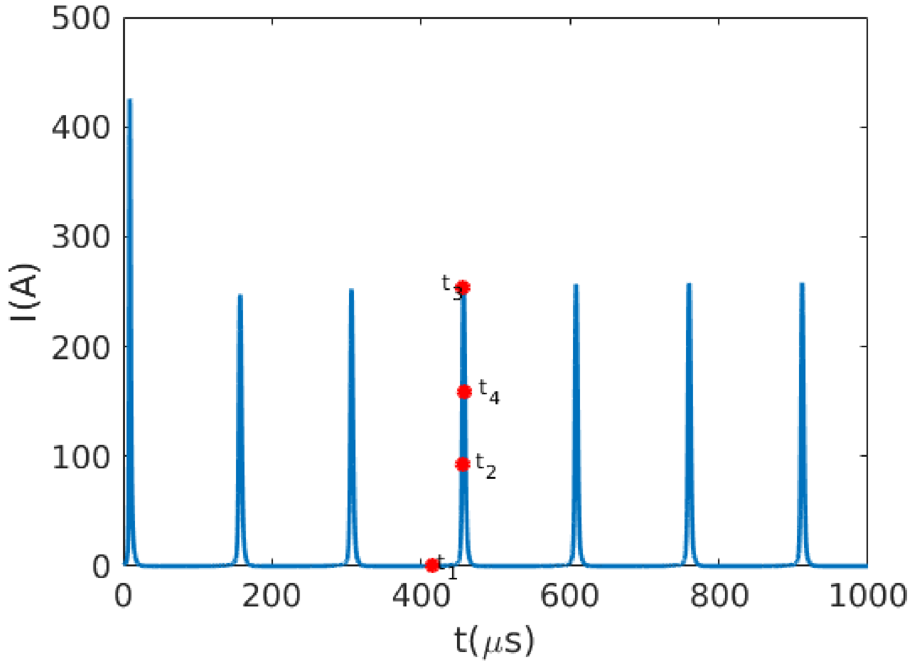
Case with damping factor for the anomalous collision frequency and varying the initial energy density.





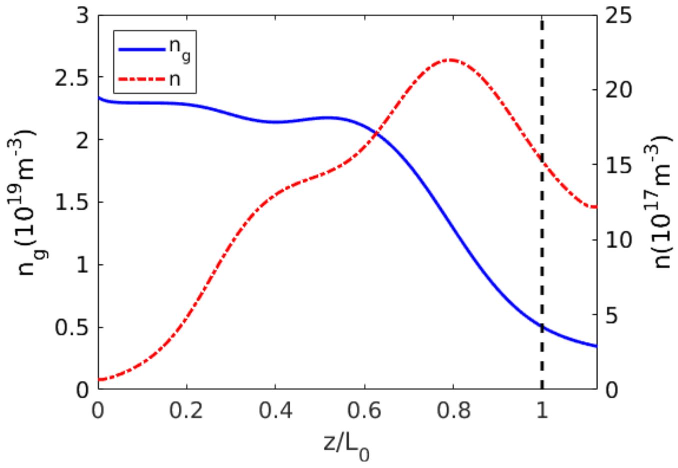
B) Results for self-consistent simulations

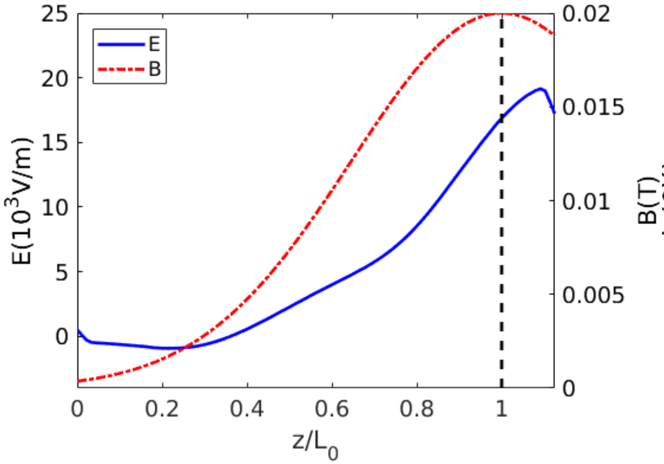
Test case: Beta = 0.2; W0 = 2e-4; V = 250 V

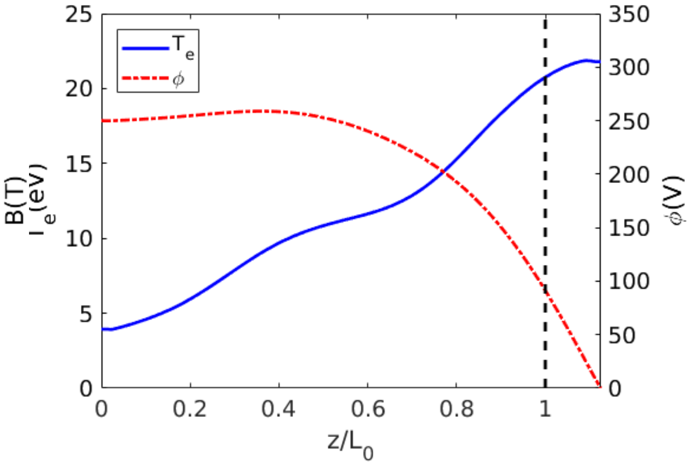


B) Results for self-consistent simulations

Test case: Beta = 0.2; W0 = 2e-4; V = 250 V
Average over the simulation time







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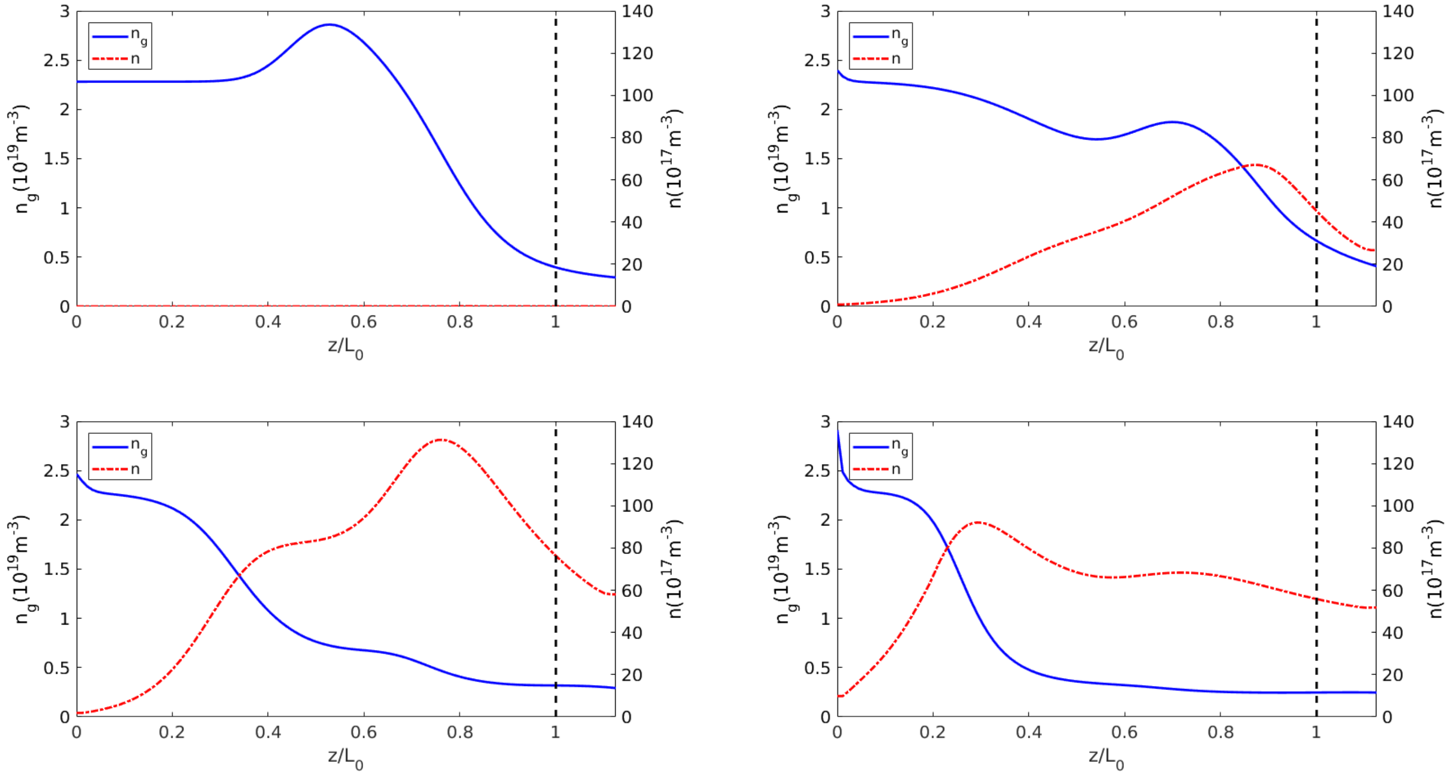
Snapshots



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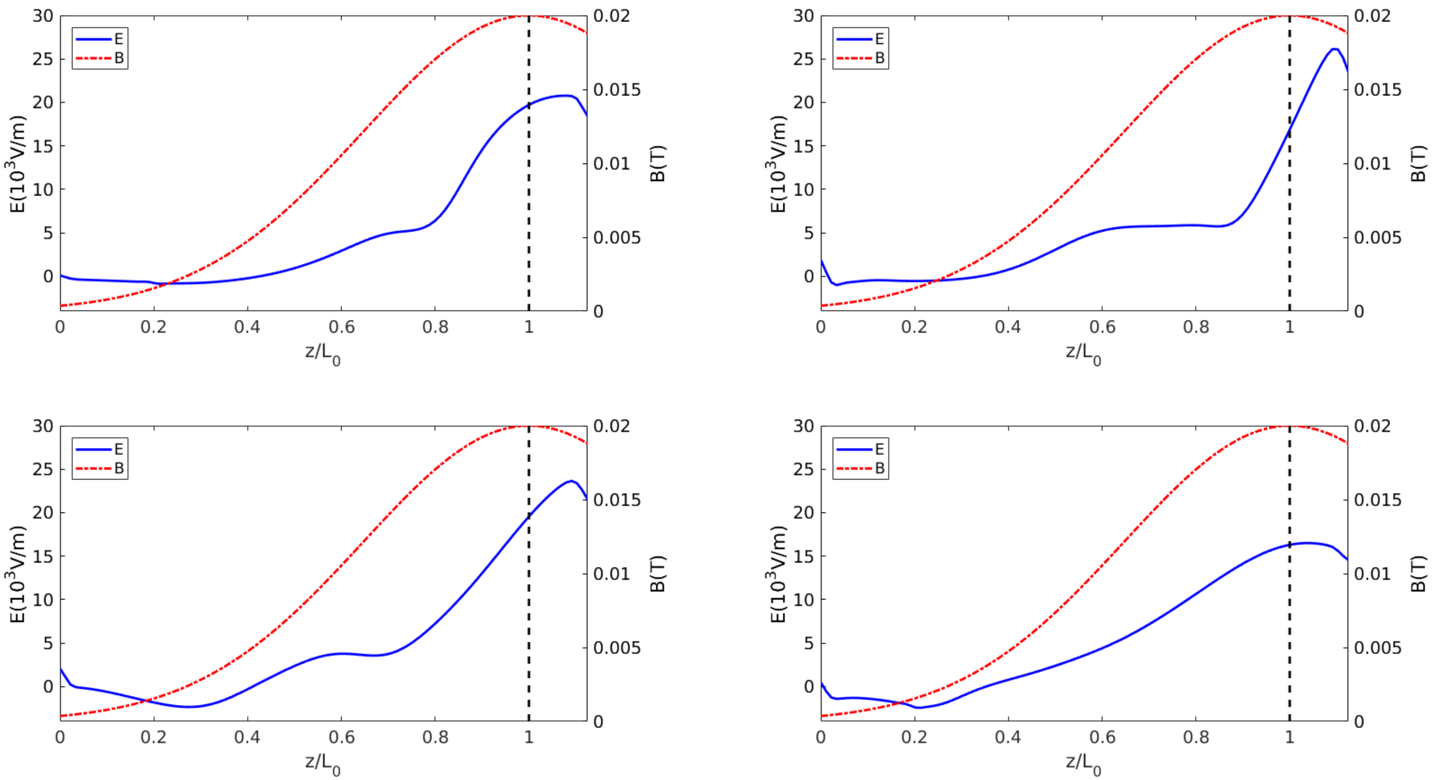

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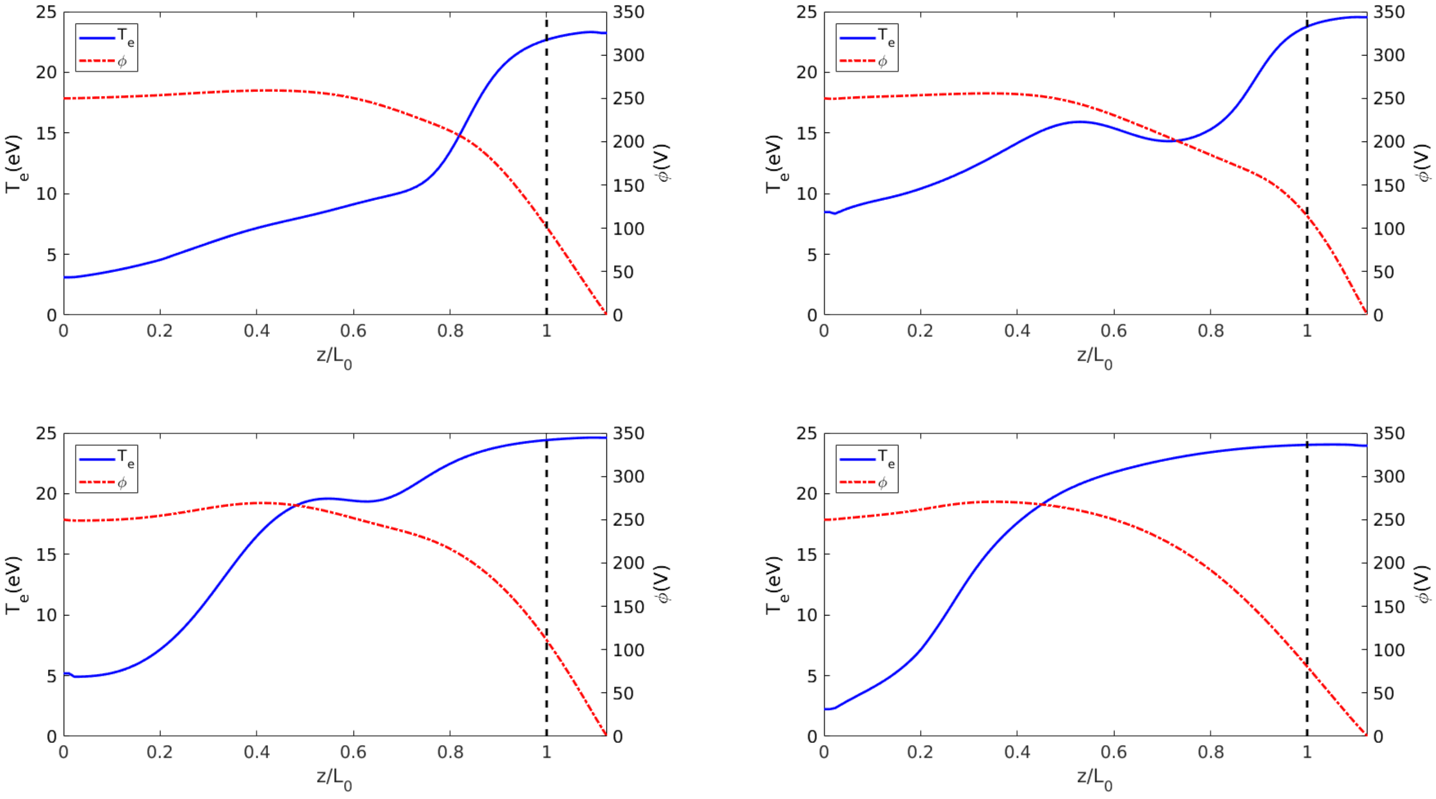
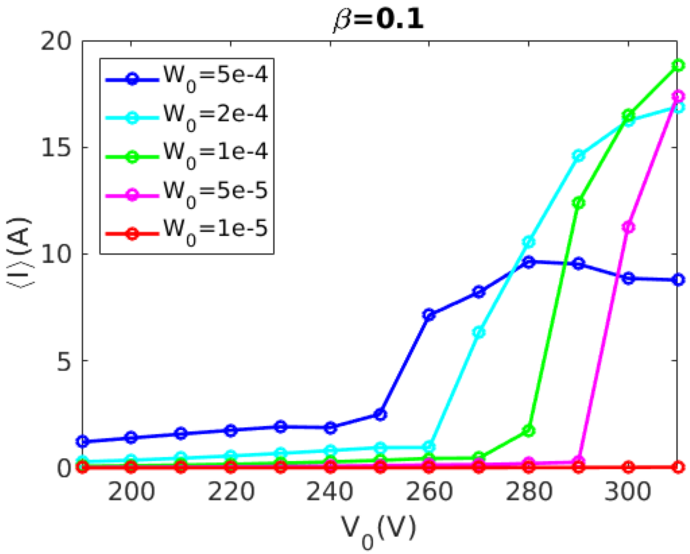
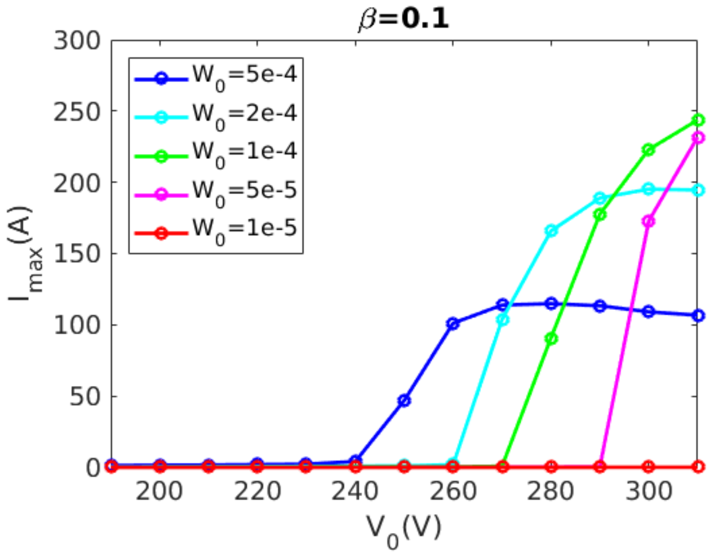


B) Results for self-consistent simulations

Test case: Beta = 0.1





B) Results for self-consistent simulations

Test case: Damping on growth rate, varying maximum energy density